Euler's Method

Euler's method is used to solve ordinary differential equations[1]. Ordinary Differential Equations have a wide application for example in dynamical and electrical systems. Euler's methods can be applied to a first order equation or to higher order equations through first resolving them to systems of first order equations. Let us consider applying Euler's method to the following first order ordinary differential equation:

$$\frac{dx}{dt} = f(t, x)$$

In any t-interval $t_{n-1} \le t \le t_n$ Euler's method advances the solution x(t) from $x_{n-1} \approx x(t_{n-1})$ to $x_n \approx x(t_n)$. Euler's method advances the solution using the relaion:

$$x_n = x_{n-1} + k\, f(t_{n-1}, x_{n-1})$$

This methods are applied to the test problems

$$\frac{dx}{dt} = t \text{ with } x(0)=0,$$

$$\frac{dx}{dt} = x \text{ with } x(0)=1.$$

which have analytic solutions $x=t^2/2$ and $x=e^t$ respectively on an Excel spreadsheet[2].

Euler's method may be applied to higher order ordinary differential equations by first converting an n^{th} order ODE to a system on n first order ODEs for example a second order ODE is converted to a system of two first order ODEs and the method can then be applied to the system[3].

Euler's method is simple, but it is a relatively inefficient method. For a more efficient class of methods the reader is referred to Runge Kutta methods[4].

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[1] Ordinary Differential Equations
[2] Solution of a first order ordinary differential equation by Euler's method - Excel Spreadsheet
[3] Applying Euler's Method to a second order ODE
[4] Runge-Kutta Methods