

Runge-Kutta Methods

Runge Kutta methods are used to solve ordinary differential equations. Ordinary Differential Equations have a wide application for example in dynamical and electrical systems. Runge-Kutta methods can be applied to a first order equation or to higher order equations through first resolving them to systems of first order equations. Let us consider applying Runge-Kutta methods to the following first order ordinary differential equation:

$$\frac{dx}{dt} = f(t, x)$$

In any t-interval $t_{n-1} \leq t \leq t_n$ the Runge-Kutta method advances the solution $x(t)$ from $x_{n-1} \approx x(t_{n-1})$ to $x_n \approx x(t_n)$. Two examples of Runge Kutta methods are given.

Second order R-K method

$$\begin{aligned} k_1 &= k f(t_{n-1}, x_{n-1}) \\ k_2 &= k f(t_{n-1} + \frac{1}{2}k, x_{n-1} + \frac{1}{2}k_1) \\ x_n &= x_{n-1} + k_2 \end{aligned}$$

Fourth order R-K method

$$\begin{aligned} k_1 &= k f(t_{n-1}, x_{n-1}) \\ k_2 &= k f(t_{n-1} + \frac{1}{2}k, x_{n-1} + \frac{1}{2}k_1) \\ k_3 &= k f(t_{n-1} + \frac{1}{2}k, x_{n-1} + \frac{1}{2}k_2) \\ k_4 &= k f(t_{n-1} + k, x_{n-1} + k_3) \\ x_n &= x_{n-1} + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} \end{aligned}$$

This methods are applied to the test problems

$$\frac{dx}{dt} = t \quad \text{with } x(0)=0,$$

$$\frac{dx}{dt} = x \quad \text{with } x(0)=1.$$

which have analytic solutions $x=t^2/2$ and $x=e^t$ respectively.

The spreadsheets can be downloaded from the webpage <http://www.numerical-methods.com/RungeKutta>